50:198:231 – Spring 2009

Homework 6 – due Mon. 27 April

(1) Write a Pep8 program which on input y (an integer with $y > 1582$) outputs "leap" if y is a leap-year, and "non-leap" otherwise.

Recall the *leap-year rule* of the Gregorian calendar:
y (> 1582) is a leap-year iff $400|y$ or $(4|y$ and $100 \not| y)$.

Use a subroutine which (for input integers n, m) sets the flag Z to 1 if $m|n$ (i.e., m divides n, i.e., $n\%m = 0$), and sets Z to 0 otherwise. Your main program should call this subroutine three times.

The pre-condition $y > 1582$ does not need to be tested.

(1.a) Explain how your program works (use a flow-chart).

(1.b) Write the program in Pep8 and test it in the Pep8 Simulator; save your program in a file while you debug. *Hand in the Pep8 Simulator results* (program, loadable machine code in hex, symbol table, test-input and -output).

(2) For the truth table in fig. 10.3 (p. 489):

(2.a) Find an OR-of-ANDs boolean formula for $x(a, b, c)$, and an OR-of-ANDs boolean formula for $y(a, b, c)$.

(2.b) Draw one circuit, with 3-variable input (a, b, c) and 2-variable output $(x(a, b, c), y(a, b, c))$.

(3) Find a circuit that uses only AND, OR, NOT gates, with 3-variable input (x_1, x_2, x_3), and 2-variable output (y_1, y_2), defined by

$$y_1 = \begin{cases} x_2 \vee \overline{x}_3 & \text{if } x_1 = 1 \ , \\ x_2 \wedge x_3 & \text{if } x_1 = 0 \ , \end{cases}$$

$$y_2 = y_1 \oplus (x_1 \wedge x_2) \ .$$

(4) Minimize the function $f(x, y, z) = \overline{x}\,\overline{y}\,z \vee x\,\overline{y}\,z \vee \overline{x}\,y\,z \vee x\,y\,z$,

(4.a) by using algebraic methods (distributivity, $s \vee \overline{s} = 1$, $s \wedge \overline{s} = 0$);

(4.b) by using a Karnaugh map.

(5) Assume that fork (a.k.a. copy, fan-out, wire-splitting), as well as wire permutations (a.k.a. wire-crossing), are available for circuit design.

(5.1) Show that $\{1, \wedge, \oplus\}$ is a universal family of gates.

(5.2) Show that $\{\wedge, \oplus\}$ is *not* universal.
(Note: Here, the constants 1 and 0 are not explicitly available; in order to obtain them they would need to be manufactured from $\{\wedge, \oplus\}$, if that's possible.)