

**Homework 2 – due Monday 16 Feb.**

(1) Let  $n$  be a non-negative integer, and let  $b_{k-1}b_{k-2}\dots b_2b_1b_0$  be its binary unsigned representation.

(1.1) Give a formula for  $k$  as a function of  $n$ , assuming there are no leading zeros.

(1.2) Give a formula for  $b_i$  as a function of  $n$  and  $i$  (for any  $i$  with  $0 \leq i \leq k-1$ ).

(2) Suppose the  $k$ -bit two's complement representation of  $z$  is  $1b_{k-2}\dots b_2b_1b_0$ . Assume  $z$  is representable (i.e.,  $-2^{k-1} \leq z \leq -1$ ).

What is the  $(k+1)$ -bit two's complement representation of that same  $z$ ? Prove your answer.

(3.1) Represent the following integers in two's complement, with a minimum number of bits: 12, -12, 1000, -999 (these are given in decimal).

**Show your method.**

(3.2) For the following two's complement representations, find the sign-magnitude decimal representation: 10111001, 0100000, 111111111, 0001110001.

(4) For two's complement representations with variable numbers of bits:

(4.1) What is the relation between  $1b_{k-2}\dots b_1b_0$  and  $1b_{k-2}\dots b_1b_00$  (i.e., a 0 has been appended on the right)?

(4.2) What is the relation between  $1b_{k-2}\dots b_1b_0$  and  $01b_{k-2}\dots b_1b_0$  (i.e., a 0 has been appended on the left)?

(4.3) What is the relation between  $0b_{k-2}\dots b_1b_0$  and  $0b_{k-2}\dots b_1b_00$  (i.e., a 0 has been appended on the right)?

(4.4) What is the relation between  $0b_{k-2}\dots b_1b_0$  and  $10b_{k-2}\dots b_1b_0$  (i.e., a 1 has been appended on the left)?

(5) For the following two's complement additions with 9 bits, find V, C, N, Z:

$$0\ 1010\ 1100 + 0\ 0011\ 1010$$

$$1\ 1110\ 0101 + 0\ 0011\ 0101$$

$$1\ 0001\ 1011 + 1\ 0101\ 0100$$

$$1\ 1000\ 0101 + 1\ 1101\ 0110$$

$$0\ 0011\ 0100 + 0\ 1101\ 0010$$

$$1\ 0010\ 0111 + 0\ 1010\ 0111$$

(6.1) For the following two's complement addition

$$1001\ 0101 + b_7b_6b_5b_4\ b_3b_2b_1b_0 = 0111\ 0010$$

we know that  $V = 1$ ,  $C = 1$ . Find  $b_7b_6b_5b_4\ b_3b_2b_1b_0$ .

Suppose we do not know V nor C. Can we still find  $b_7b_6b_5b_4\ b_3b_2b_1b_0$ ?